Government College of Engineering and Research
Avasari, Pune

Fundamental of Finite Element Analysis

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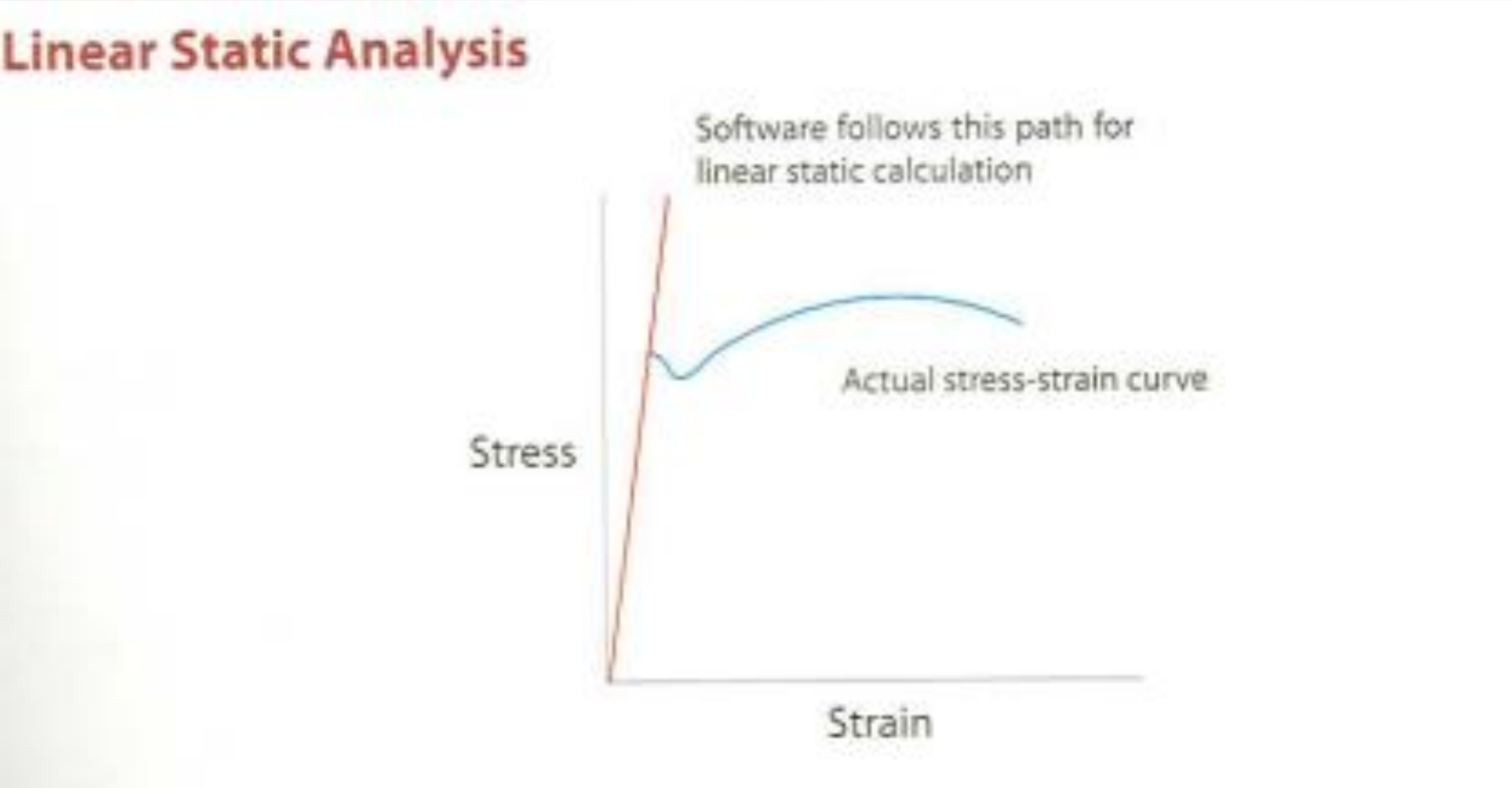
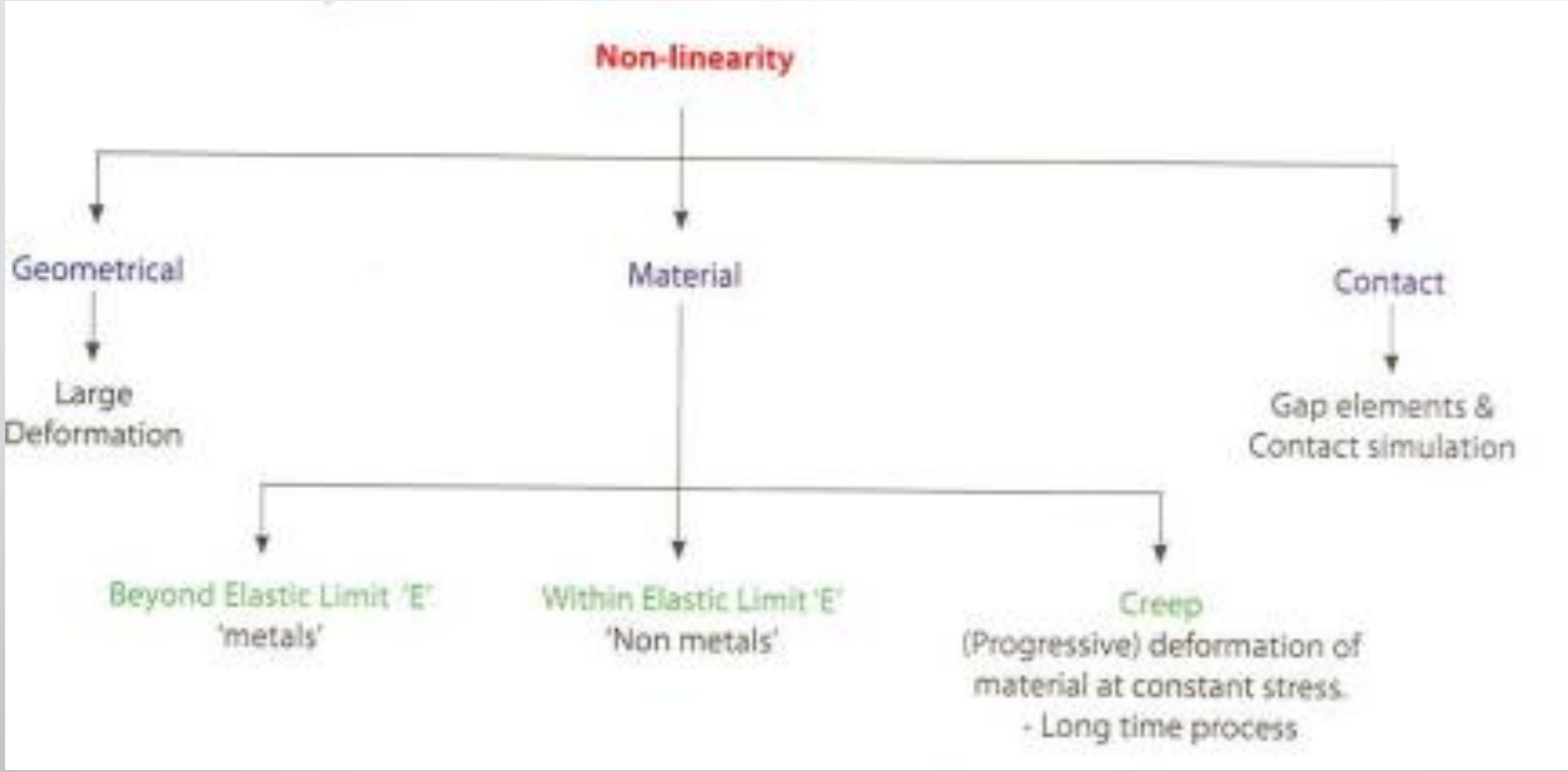
Unit 2

1D Elements

Linear and non Linear statics analysis

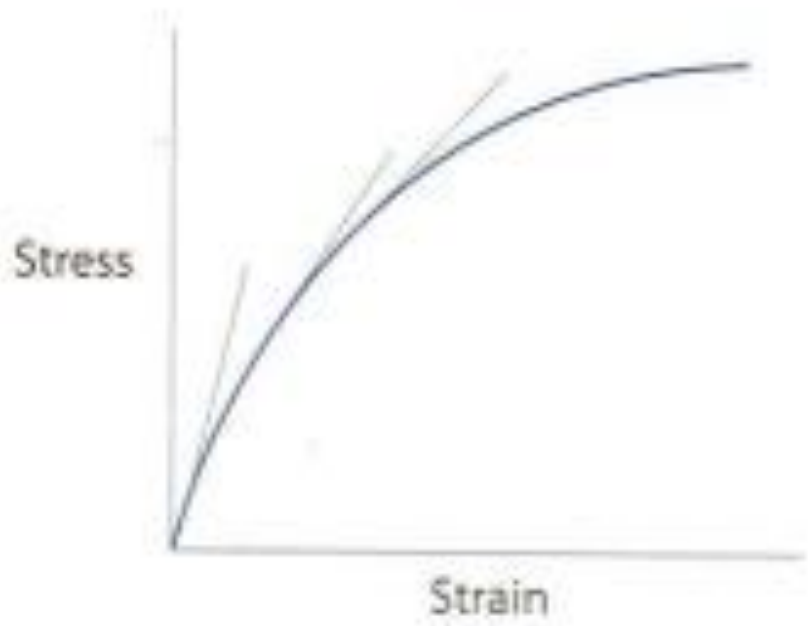


Non Linear analysis

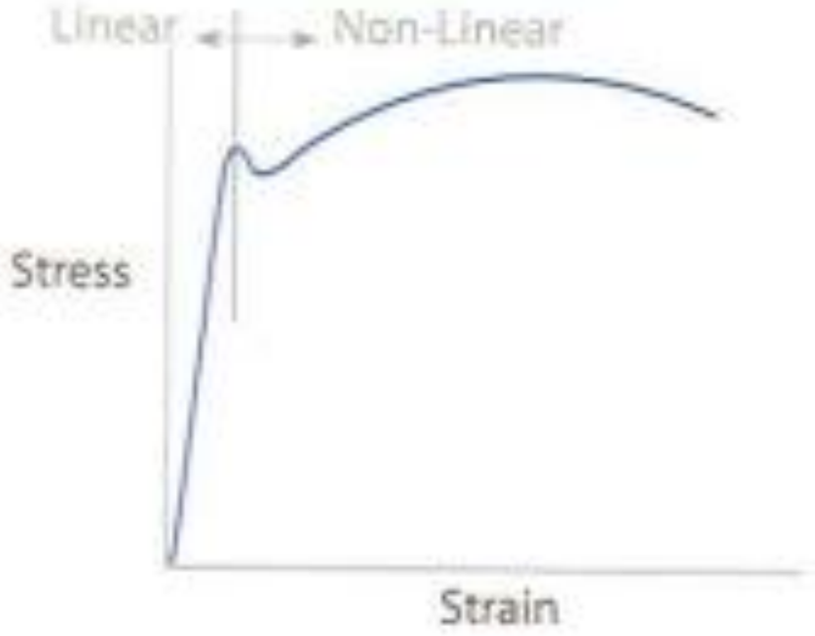


Material based non linearity:

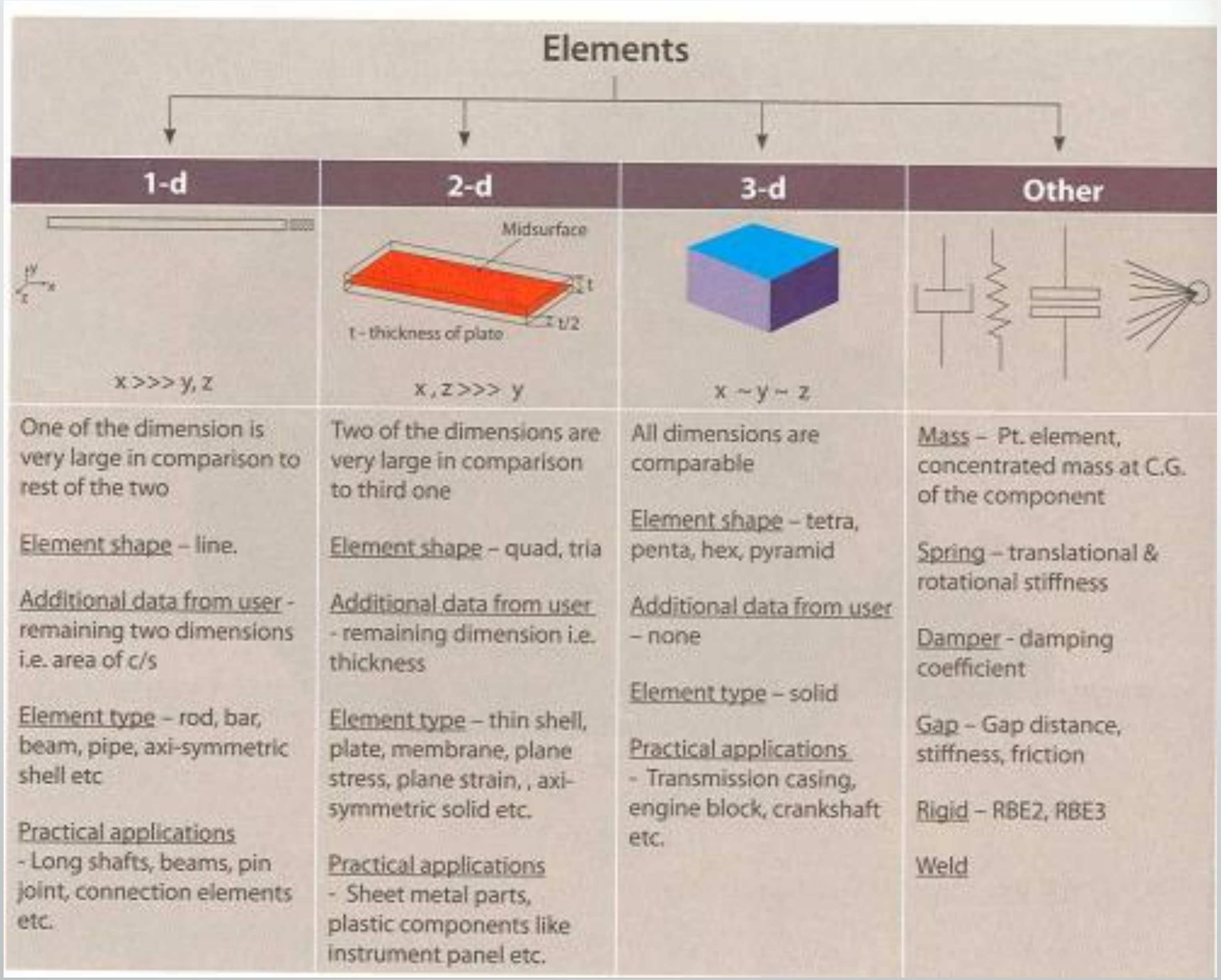
Within E (non-metal)



Beyond E (metals)



Type of Element

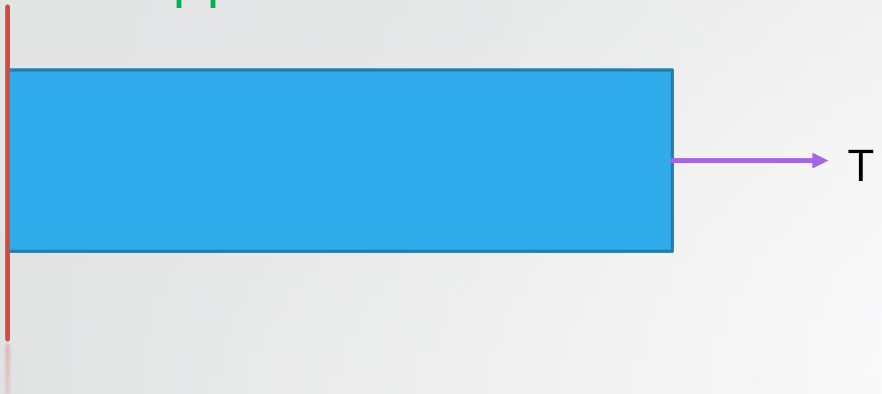


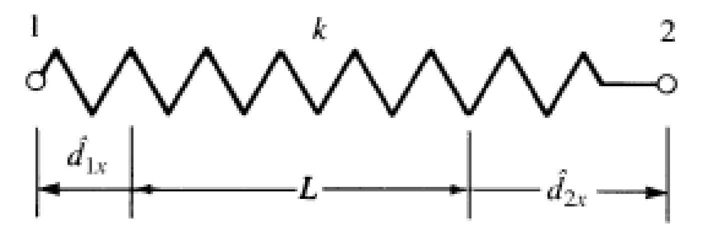
Elements

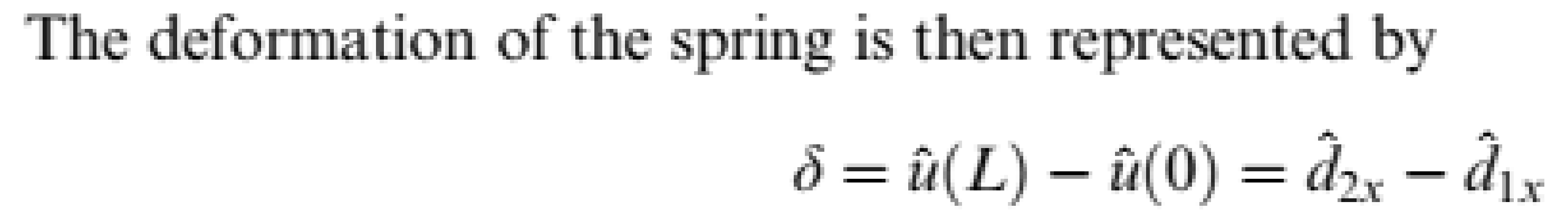
1-d	2-d	3-d	Other
x >>> y, z	x , z >>> y	x ~ y ~ z	
One of the dimension is very large in comparison to rest of the two Element shape – line. Additional data from user - remaining two dimensions i.e. area of c/s Element type – rod, bar, beam, pipe, axi-symmetric shell etc Practical applications - Long shafts, beams, pin joint, connection elements etc.	Two of the dimensions are very large in comparison to third one Element shape – quad, tria Additional data from user - remaining dimension i.e. thickness Element type – thin shell, plate, membrane, plane stress, plane strain, , axi-symmetric solid etc. Practical applications - Sheet metal parts, plastic components like instrument panel etc.	All dimensions are comparable Element shape – tetra, penta, hex, pyramid Additional data from user – none Element type – solid Practical applications - Transmission casing, engine block, crankshaft etc.	Mass – Pt. element, concentrated mass at C.G. of the component Spring – translational & rotational stiffness Damper - damping coefficient Gap – Gap distance, stiffness, friction Rigid – RBE2, RBE3 Weld

Stiffness matrix for Bar Problem

1)Direct Approach

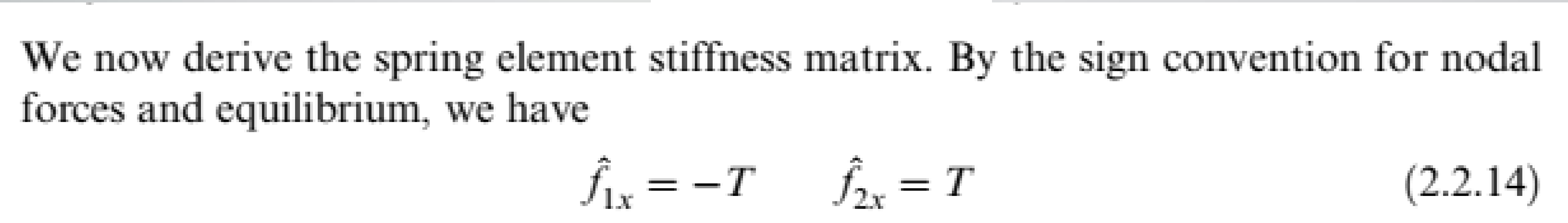





The deformation of the spring is then represented by

$$\delta = \hat{u}(L) - \hat{u}(0) = \hat{d}_{2x} - \hat{d}_{1x}$$


$$T = k\delta$$


We now derive the spring element stiffness matrix. By the sign convention for nodal forces and equilibrium, we have

$$\hat{f}_{1x} = -T \qquad \hat{f}_{2x} = T \tag{2.2.14}$$

$$T = -\hat{f}_{1x} = k(\hat{d}_{2x} - \hat{d}_{1x})$$

$$T = \hat{f}_{2x} = k(\hat{d}_{2x} - \hat{d}_{1x})$$

Forces at nodal point

$$\hat{f}_{1x} = k(\hat{d}_{1x} - \hat{d}_{2x})$$
$$\hat{f}_{2x} = k(\hat{d}_{2x} - \hat{d}_{1x})$$

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

Rayleigh –Ritz (Variation) Method

The principle of Minimum Potential Energy:

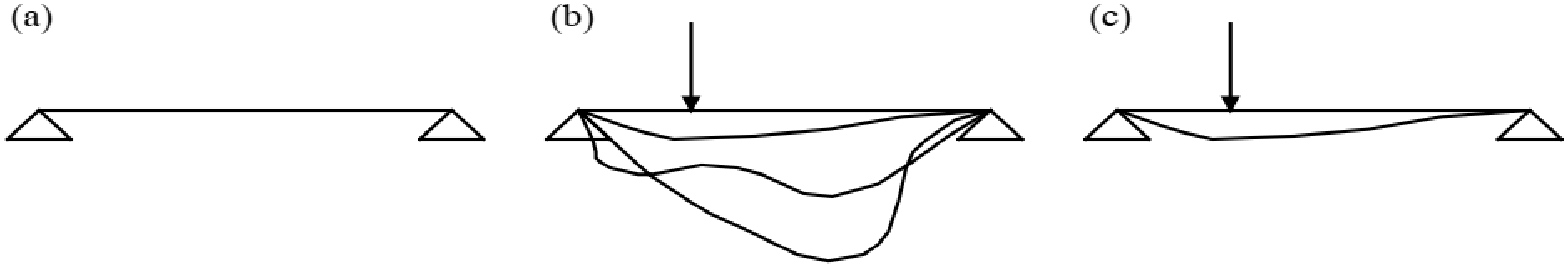


Figure 1 The notion of equilibrium deformed state of a pinned-pinned beam

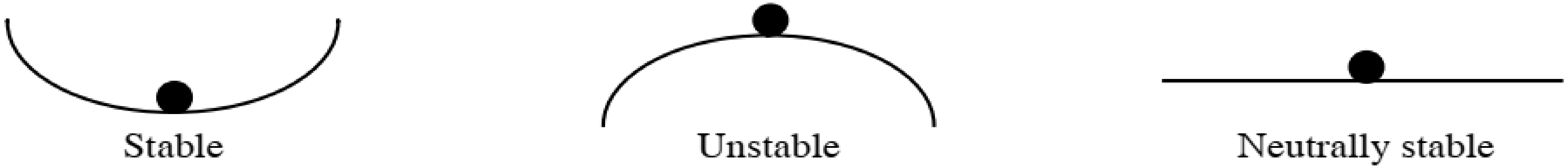
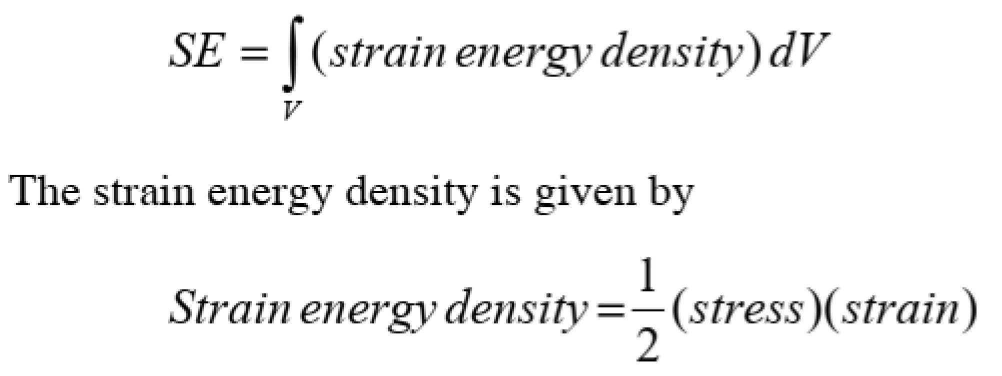

Stable Unstable Neutrally stable

Figure 2 Three equilibrium states of a rolling ball

$PE = SE + WP$



$$SE = \int_V (strain\,energy\,density)\,dV$$

The strain energy density is given by

$$Strain\,energy\,density = \frac{1}{2}(stress)(strain)$$

$WP = F*u$

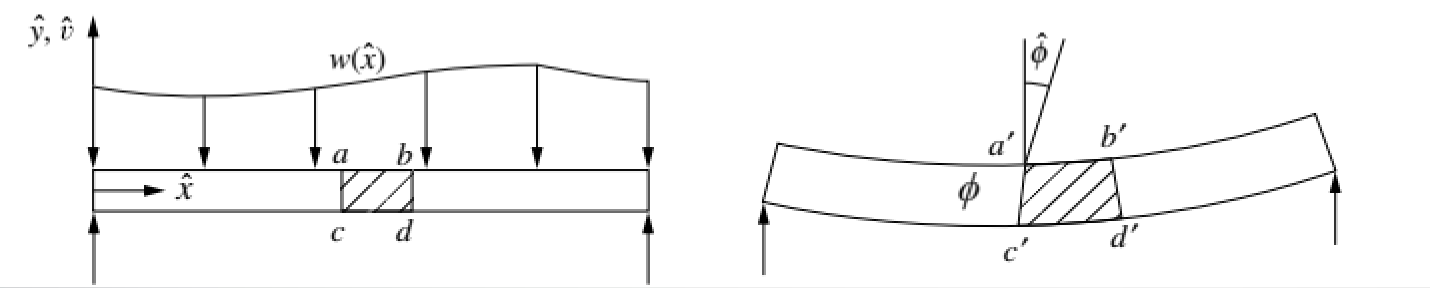
The principle of Minimum Potential Energy:

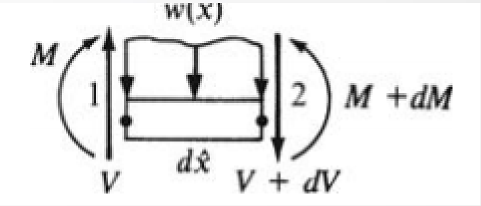
For conservative structural systems, of all the kinematically admissible deformations, those corresponding to the equilibrium state extremize (i.e., minimize or maximize) the total potential energy. If the extremum is a minimum, the equilibrium state is stable.

Stiffness matrix of Beam element

1)Direct Approach

- A beam is a long, slender structural member generally subjected to transverse loading that produces significant bending effects
- Hence, the degrees of freedom considered per node are a transverse displacement and a rotation



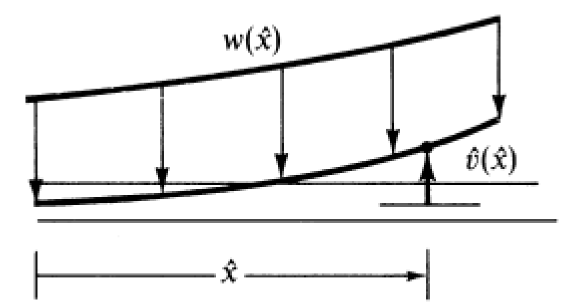


- The beam is of length L with
- axial local coordinate x and transverse local coordinate y.
- The local transverse nodal displacements are given by d, and the rotations by ϕ
- The local nodal forces are given by fiy's and the bending moments by mi's as shown.
- V is shear force
- initially neglect all axial effects

Using Euler-Bernouli Beam Theory

$$w = -\frac{dV}{d\hat{x}} \qquad V = \frac{dM}{d\hat{x}}$$

$\hat{v}$ is the transverse displacement function in the y direction



(a) Portion of deflected curve of beam

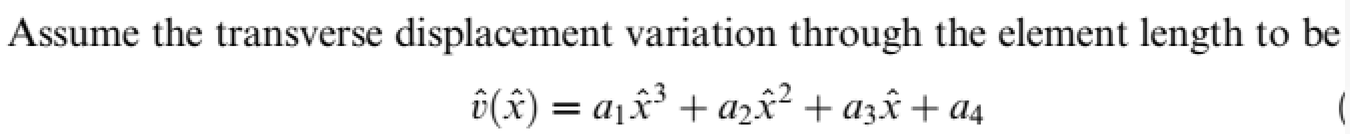
$$\hat{\phi} = d\hat{v}/d\hat{x}$$

$$\frac{d^2\hat{v}}{d\hat{x}^2} = \frac{M}{EI}$$

$$\hat{V} = EI\frac{d^3\hat{v}}{d\hat{x}^3}$$

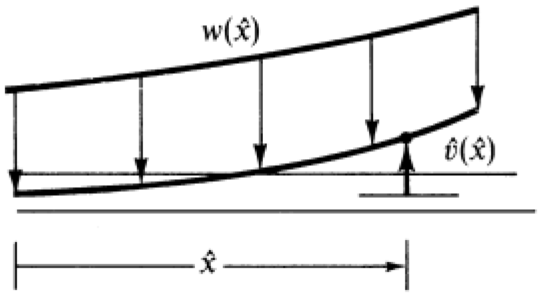
$$EI\frac{d^4\hat{v}}{d\hat{x}^4} = 0$$

Selection of displacement Function



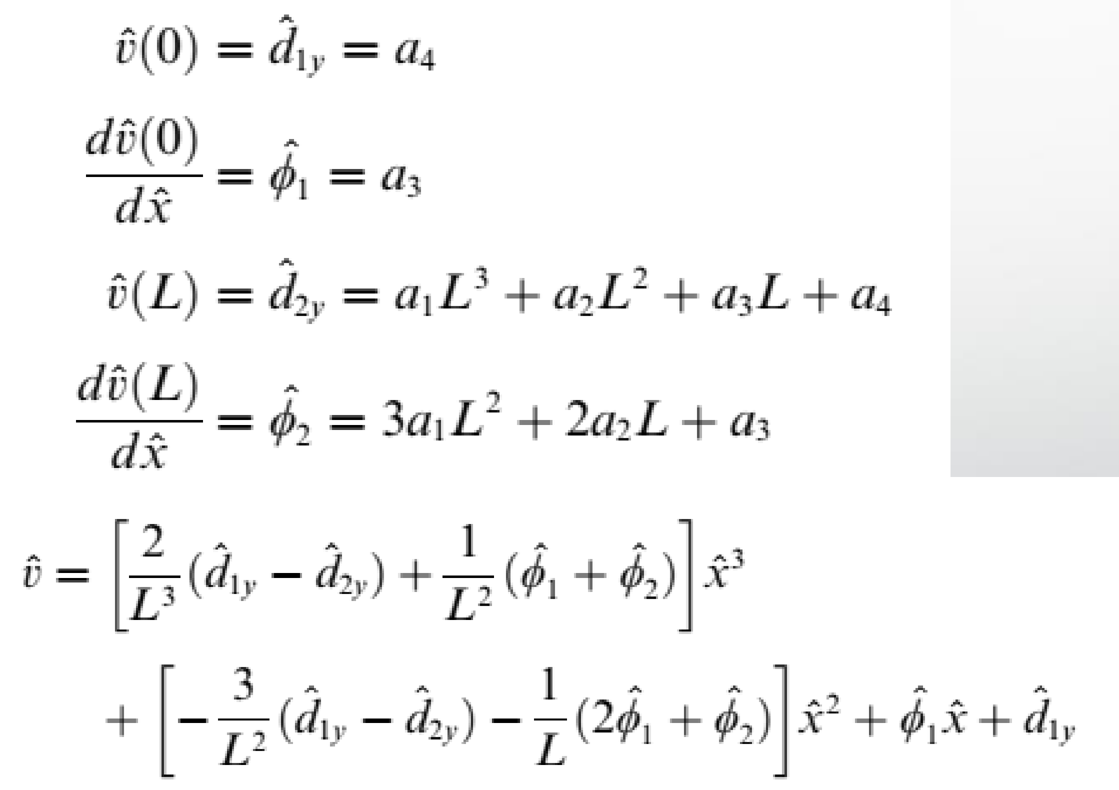
Assume the transverse displacement variation through the element length to be

$$\hat{v}(\hat{x}) = a_1\hat{x}^3 + a_2\hat{x}^2 + a_3\hat{x} + a_4$$



(a) Portion of deflected curve of beam

Using the boundary conditions



$$\hat{v}(0) = \hat{d}_{1y} = a_4$$

$$\frac{d\hat{v}(0)}{d\hat{x}} = \hat{\phi}_1 = a_3$$

$$\hat{v}(L) = \hat{d}_{2y} = a_1L^3 + a_2L^2 + a_3L + a_4$$

$$\frac{d\hat{v}(L)}{d\hat{x}} = \hat{\phi}_2 = 3a_1L^2 + 2a_2L + a_3$$

$$\hat{v} = \left[\frac{2}{L^3}(\hat{d}_{1y} - \hat{d}_{2y}) + \frac{1}{L^2}(\hat{\phi}_1 + \hat{\phi}_2)\right]\hat{x}^3 + \left[-\frac{3}{L^2}(\hat{d}_{1y} - \hat{d}_{2y}) - \frac{1}{L}(2\hat{\phi}_1 + \hat{\phi}_2)\right]\hat{x}^2 + \hat{\phi}_1\hat{x} + \hat{d}_{1y}$$

Displacement function in terms of d, and ϕ

To, derive the element stiffness matrix and equations using a direct equilibrium approach. Now relate the nodal and beam theory sign conventions for shear forces and bending moments.

$$\hat{f}_{1y} = \hat{V} = EI\frac{d^3\hat{v}(0)}{d\hat{x}^3} = \frac{EI}{L^3}(12\hat{d}_{1y} + 6L\hat{\phi}_1 - 12\hat{d}_{2y} + 6L\hat{\phi}_2)$$

$$\hat{m}_1 = -\hat{m} = -EI\frac{d^2\hat{v}(0)}{d\hat{x}^2} = \frac{EI}{L^3}(6L\hat{d}_{1y} + 4L^2\hat{\phi}_1 - 6L\hat{d}_{2y} + 2L^2\hat{\phi}_2)$$

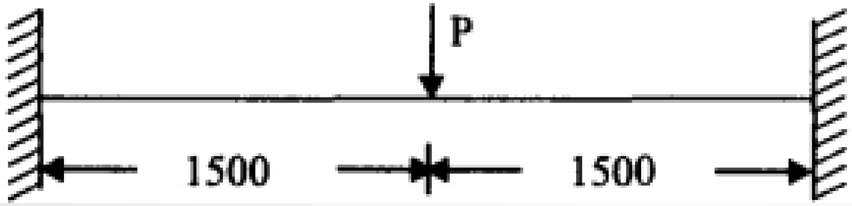
$$\hat{f}_{2y} = -\hat{V} = -EI\frac{d^3\hat{v}(L)}{d\hat{x}^3} = \frac{EI}{L^3}(-12\hat{d}_{1y} - 6L\hat{\phi}_1 + 12\hat{d}_{2y} - 6L\hat{\phi}_2)$$

$$\hat{m}_2 = \hat{m} = EI\frac{d^2\hat{v}(L)}{d\hat{x}^2} = \frac{EI}{L^3}(6L\hat{d}_{1y} + 2L^2\hat{\phi}_1 - 6L\hat{d}_{2y} + 4L^2\hat{\phi}_2)$$

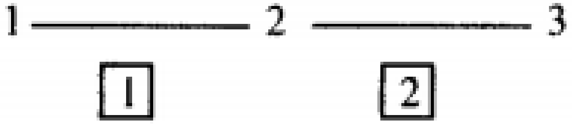
In matrix form,

$$\begin{Bmatrix} \hat{f}_{1y} \\ \hat{m}_1 \\ \hat{f}_{2y} \\ \hat{m}_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1y} \\ \hat{\phi}_1 \\ \hat{d}_{2y} \\ \hat{\phi}_2 \end{Bmatrix}$$

A concentrated load P = 50 kN is applied at the center of a fixed beam of length 3m, depth 200 mm and width 120 mm. Calculate the deflection and slope at the mid point. Assume $E = 2 \times 10^5$ N/mm^2.



The finite element model consists of 2 beam elements, as shown here, with nodes 1 and 3 at the two fixed supports and node 2 at the location where load P is applied.



Stiffness matrices of elements 1 and 2 (connected by nodes 1 and 2 ; 2 and 3 respectively, each with L = 1500 mm) are given by,

$$[K]=\frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$K_1 = \frac{2\times10^5\times\frac{(120\times200^3)}{12}}{L^3}\left[\begin{array}{ccccc} w1 & \Phi1 & w2 & \Phi2 & \\ 12 & 6L & -12 & 6L & w1 \\ 6L & 4L^2 & -6L & 2L^2 & \Phi1 \\ -12 & -6L & 12 & -6L & w2 \\ 6L & 2L^2 & -6L & 4L^2 & \Phi2 \end{array}\right]$$

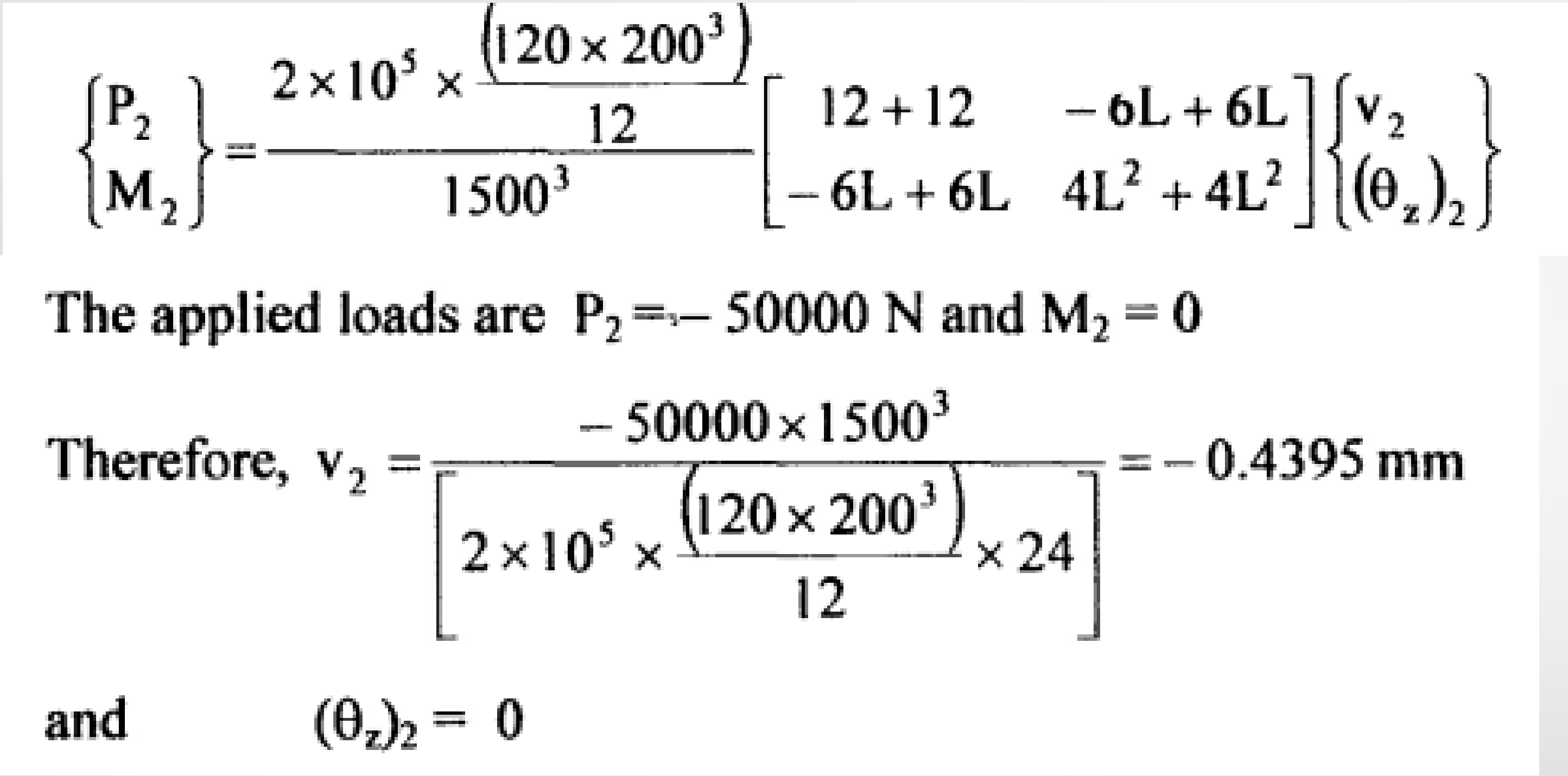
$$K_2 = \frac{2\times10^5\times\frac{(120\times200^3)}{12}}{L^3}\left[\begin{array}{ccccc} w12 & \Phi2 & w3 & \Phi3 & \\ 12 & 6L & -12 & 6L & w2 \\ 6L & 4L^2 & -6L & 2L^2 & \Phi2 \\ -12 & -6L & 12 & -6L & w3 \\ 6L & 2L^2 & -6L & 4L^2 & \Phi3 \end{array}\right]$$

$$= \frac{2\times10^5 \times \frac{120\times200^3}{12}}{1500^3} \left[\begin{array}{cccccc|c} w1 & \Phi1 & w2 & \Phi2 & w3 & \Phi3 & \\ \hline 12 & 6L & -12 & 6L & 0 & 0 & w1 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & \Phi1 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L & w2 \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 & \Phi2 \\ 0 & 0 & -12 & -6L & 12 & -16L & w3 \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 & \Phi3 \end{array}\right]$$

$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \\ P_3 \\ M_3 \end{Bmatrix} = \frac{2\times10^5\times\frac{120\times200^3}{12}}{1500^3} \left[\begin{array}{cccccc|c} w1 & \Phi1 & w2 & \Phi2 & w3 & \Phi3 & \\ 12 & 6L & -12 & 6L & 0 & 0 & w1 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & \Phi1 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L & w2 \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 & \Phi2 \\ 0 & 0 & -12 & -6L & 12 & -16L & w3 \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 & \Phi3 \end{array}\right] \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{Bmatrix}$$

Boundary conditions at fixed support w1 =w3= Φ1= Φ3 =0

$$\begin{Bmatrix} P_1 \\ M_1 \\ P_2 \\ M_2 \\ P_3 \\ M_3 \end{Bmatrix} = \frac{2\times10^5 \times \frac{120\times200^3}{12}}{1500^3} \begin{array}{cccccc|c} w1 & \Phi1 & w2 & \Phi2 & w3 & \Phi3 & \\ \hline 12 & 6L & -12 & 6L & 0 & 0 & w1 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 & \Phi1 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L & w2 \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 & \Phi2 \\ 0 & 0 & -12 & -6L & 12 & -16L & w3 \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 & \Phi3 \end{array} \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{Bmatrix}$$

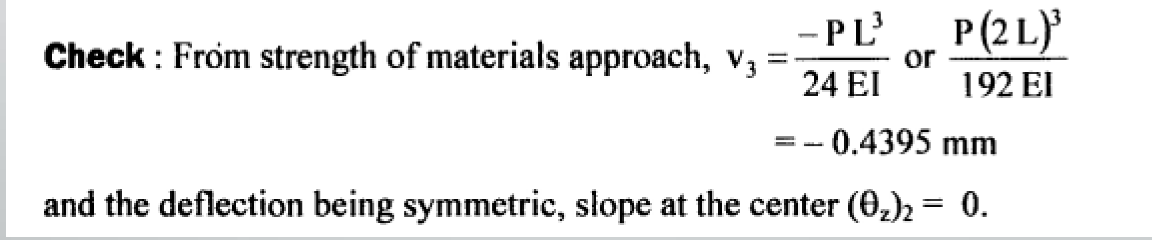


$$\begin{Bmatrix} P_2 \\ M_2 \end{Bmatrix} = \frac{2\times10^5 \times \frac{(120\times200^3)}{12}}{1500^3} \begin{bmatrix} 12+12 & -6L+6L \\ -6L+6L & 4L^2+4L^2 \end{bmatrix} \begin{Bmatrix} v_2 \\ (\theta_z)_2 \end{Bmatrix}$$

The applied loads are $P_2 = -50000$ N and $M_2 = 0$

Therefore, $v_2 = \dfrac{-50000\times1500^3}{\left[2\times10^5 \times \dfrac{(120\times200^3)}{12} \times 24\right]} = -0.4395$ mm

and $(\theta_z)_2 = 0$



Check : From strength of materials approach, $v_3 = \dfrac{-P L^3}{24 EI}$ or $\dfrac{P(2L)^3}{192 EI}$

$= -0.4395$ mm

and the deflection being symmetric, slope at the center $(\theta_z)_2 = 0$.

Stiffness matrix of Truss element

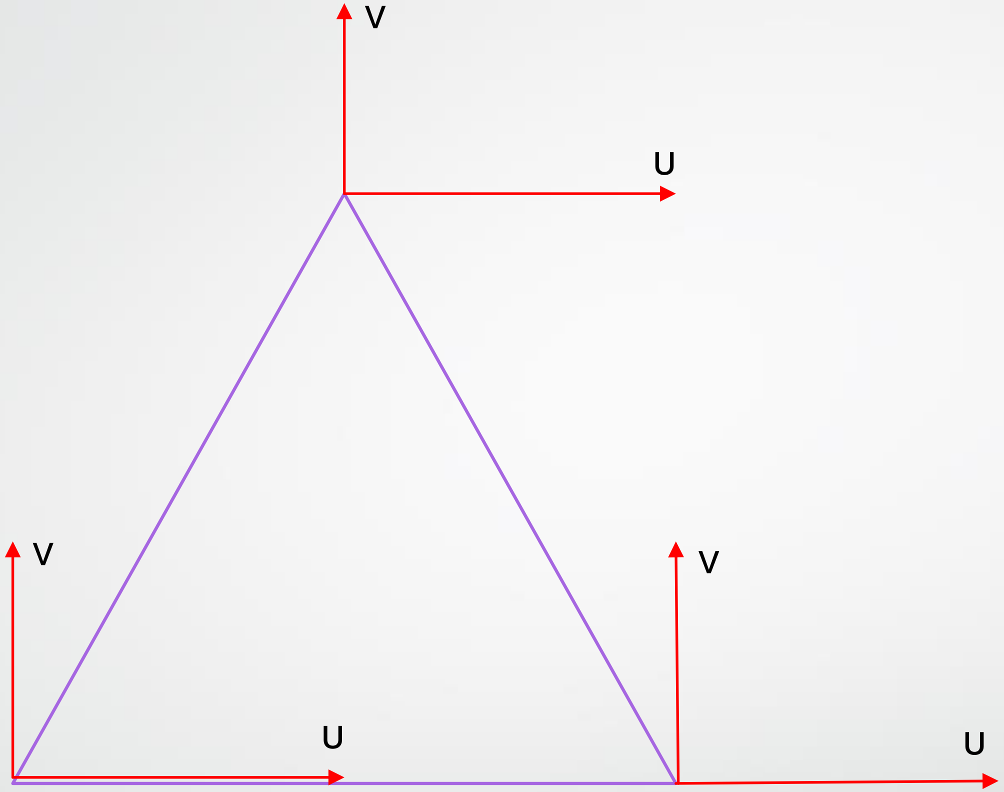
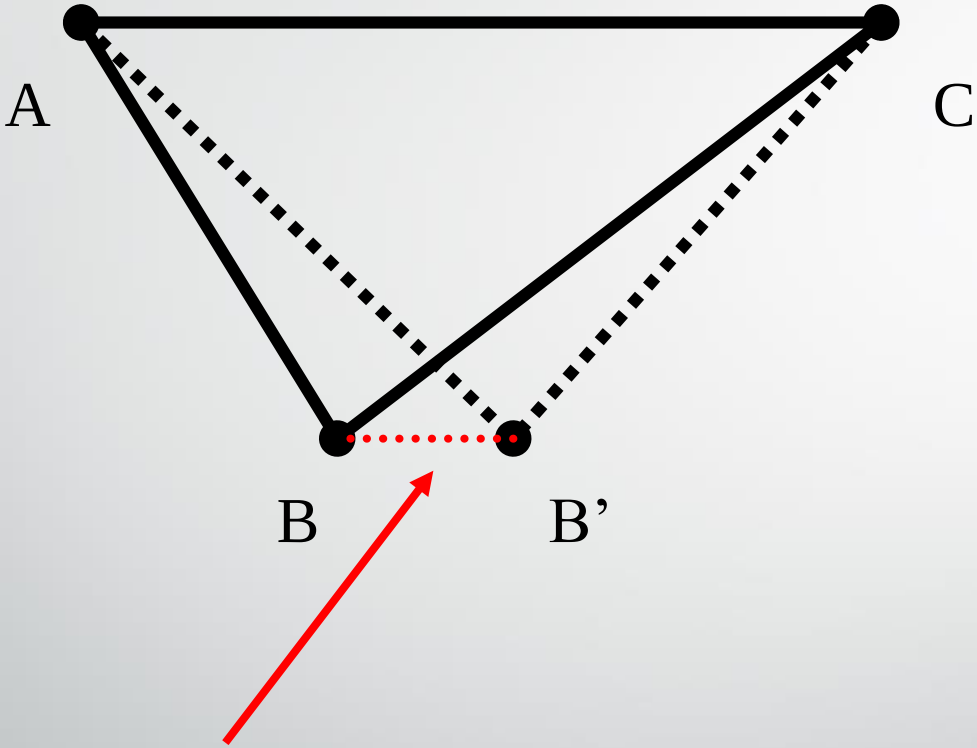
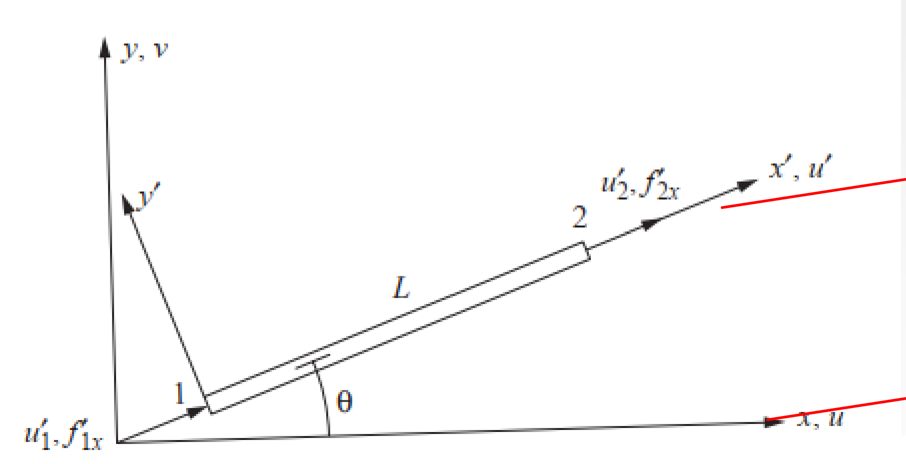


Figure shows the two dimensional plane truss. Such trusses are analysed by method of join. In FEA trusses are assumed 1D bar element



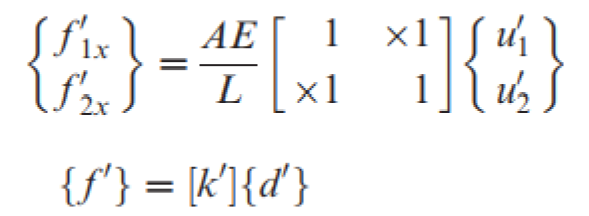
Unit Displacement



Local coordinate system

Global coordinate system

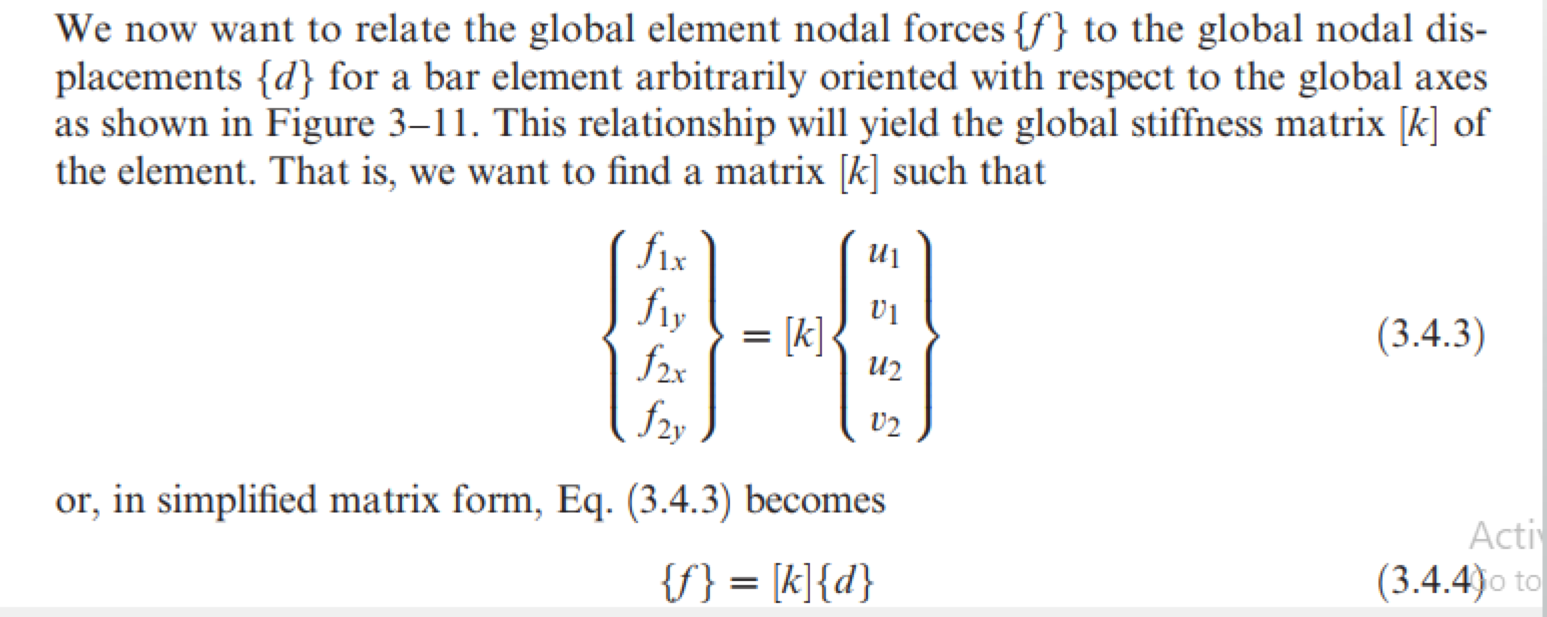
Local stiffness matrix of bar element =



$$\begin{Bmatrix} f'_{1x} \\ f'_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & \times 1 \\ \times 1 & 1 \end{bmatrix}\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix}$$

$$\{f'\} = [k']\{d'\}$$

Eq.1



We now want to relate the global element nodal forces $\{f\}$ to the global nodal displacements $\{d\}$ for a bar element arbitrarily oriented with respect to the global axes as shown in Figure 3–11. This relationship will yield the global stiffness matrix $[k]$ of the element. That is, we want to find a matrix $[k]$ such that

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix} = [k]\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \qquad (3.4.3)$$

or, in simplified matrix form, Eq. (3.4.3) becomes

$$\{f\} = [k]\{d\} \qquad (3.4.4)$$

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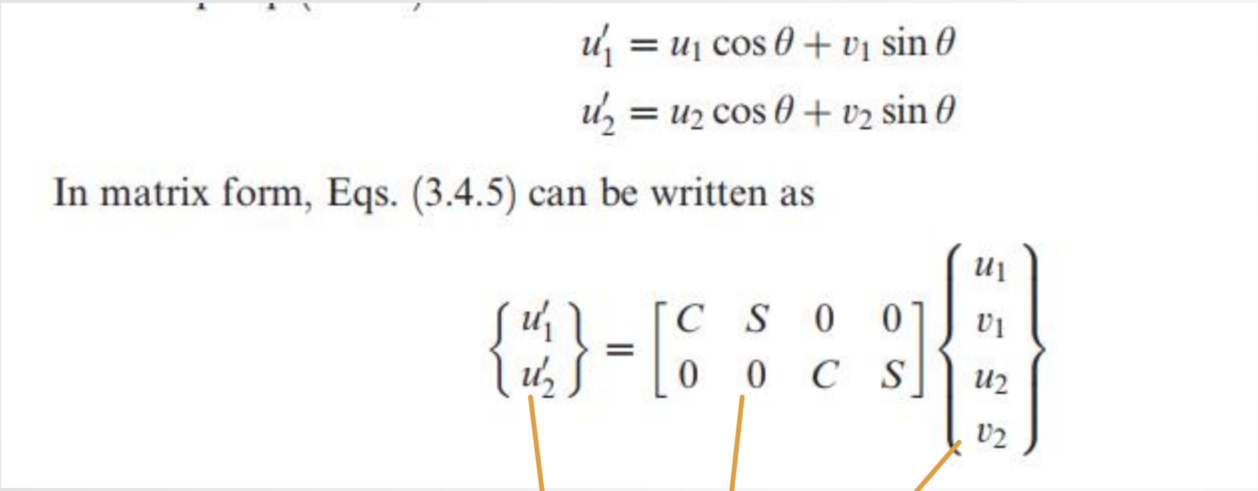
Using the relationship between local and global coordinate system



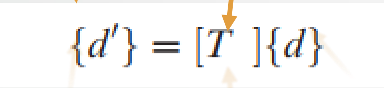
$$u'_1 = u_1 \cos\theta + v_1 \sin\theta$$

$$u'_2 = u_2 \cos\theta + v_2 \sin\theta$$

In matrix form, Eqs. (3.4.5) can be written as

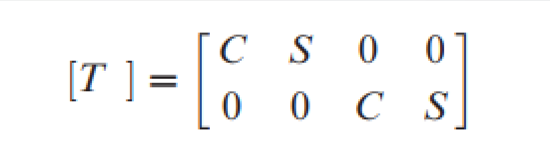
$$\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix} = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

The global displacement vector is

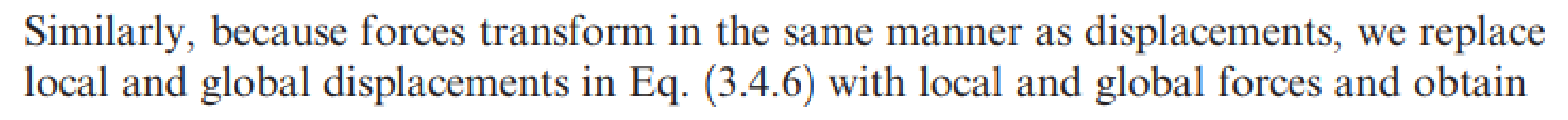


$$\{d'\} = [T\ ]\{d\}$$

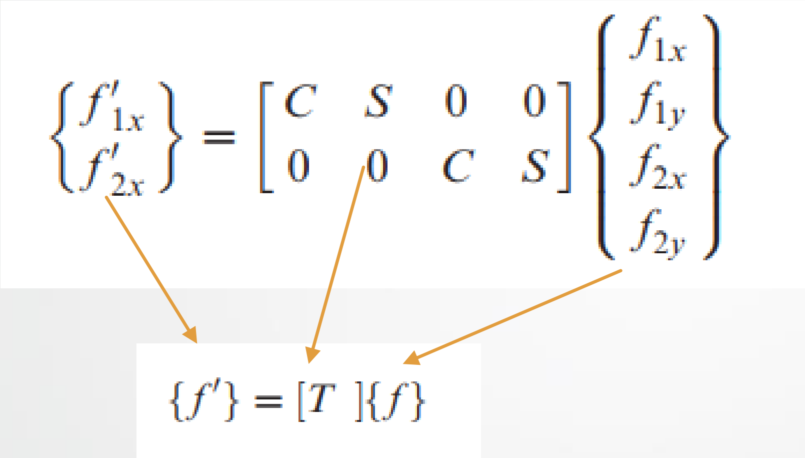
Transformation matrix=



$$[T\ ] = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}$$



Similarly, because forces transform in the same manner as displacements, we replace local and global displacements in Eq. (3.4.6) with local and global forces and obtain



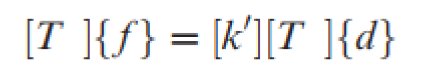
$$\begin{Bmatrix} f'_{1x} \\ f'_{2x} \end{Bmatrix} = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix} \begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix}$$

$$\{f'\} = [T\ ]\{f\}$$

By putting global displacement matrix in equation 1

$$\{f'\} = [k'][T\ ]\{d\}$$

By putting global force matrix in above equation



$$[T\ ]\{f\} = [k'][T\ ]\{d\}$$

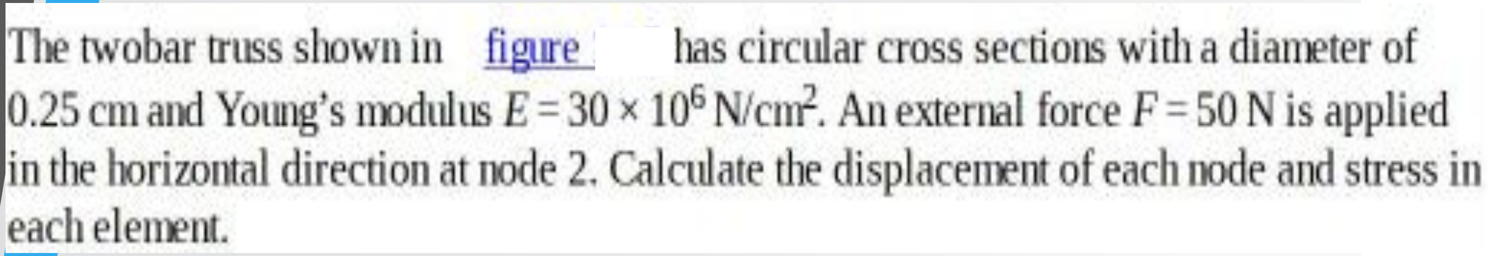
$$\{f\} = [T]^T[k'][T]\{d\}$$

$$[k] = [T]^T[k'][T]$$

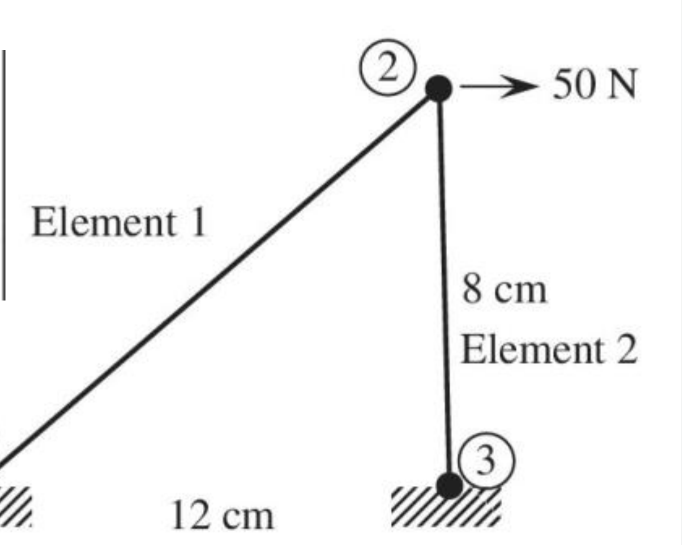
Stiffness matrix of truss element

$$[k] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & \times C^2 & \times CS \\ & S^2 & \times CS & \times S^2 \\ & & C^2 & CS \\ \text{Symmetry} & & & S^2 \end{bmatrix}$$

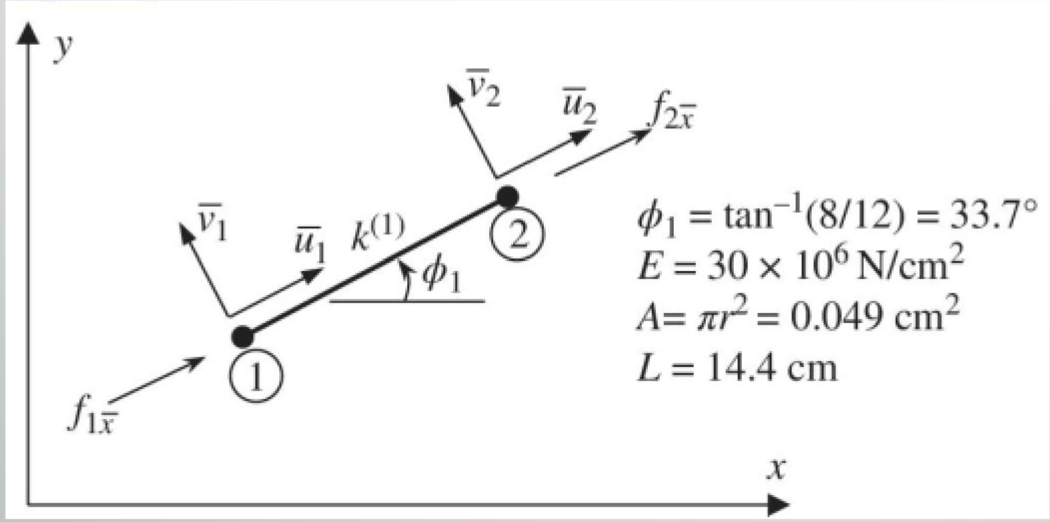
Problem:



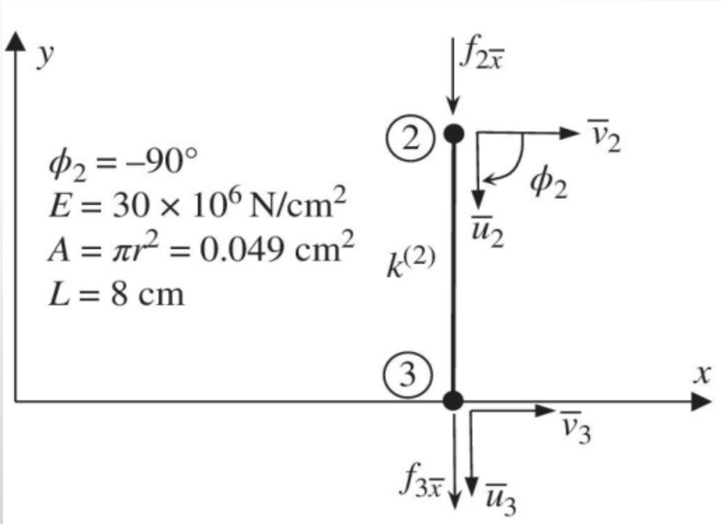
The twobar truss shown in figure has circular cross sections with a diameter of 0.25 cm and Young's modulus $E = 30 \times 10^6$ N/cm^2. An external force $F = 50$ N is applied in the horizontal direction at node 2. Calculate the displacement of each node and stress in each element.



Solution:



Element 1



Element 2

To find the stiffness matrix of each element

$$\left\{\mathbf{f}^{(1)}\right\} = \left[\mathbf{k}^{(1)}\right]\left\{\mathbf{q}^{(1)}\right\}.$$

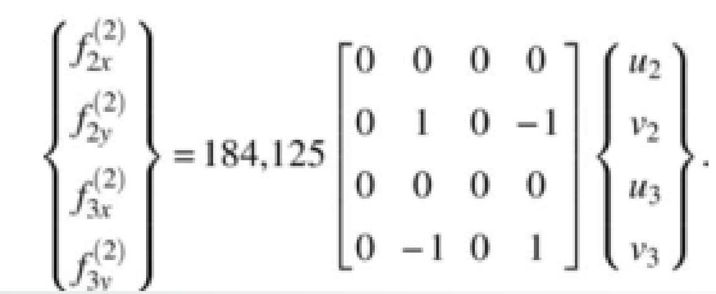
Where [k] is,

$$[k] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & \times C^2 & \times CS \\ & S^2 & \times CS & \times S^2 \\ & & C^2 & CS \\ \text{Symmetry} & & & S^2 \end{bmatrix}$$

For element 1

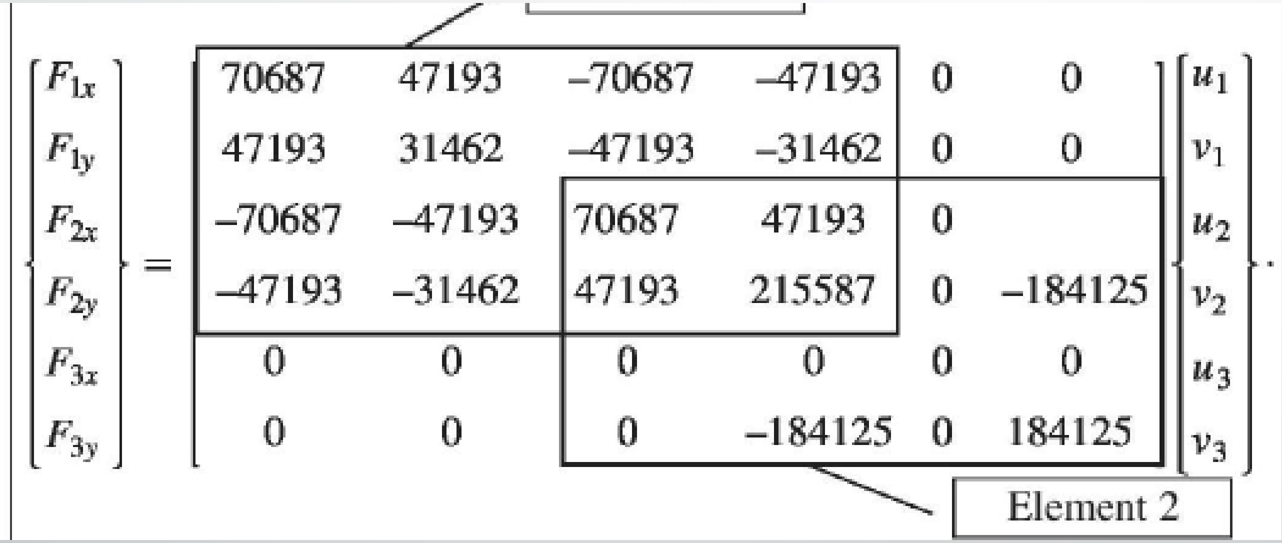
$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{1y}^{(1)} \\ f_{2x}^{(1)} \\ f_{2y}^{(1)} \end{Bmatrix} = 102{,}150\begin{bmatrix} 0.692 & 0.462 & -0.692 & -0.462 \\ 0.462 & 0.308 & -0.462 & -0.308 \\ -0.692 & -0.462 & 0.692 & 0.462 \\ -0.462 & -0.308 & 0.462 & 0.308 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}.$$

For element 2

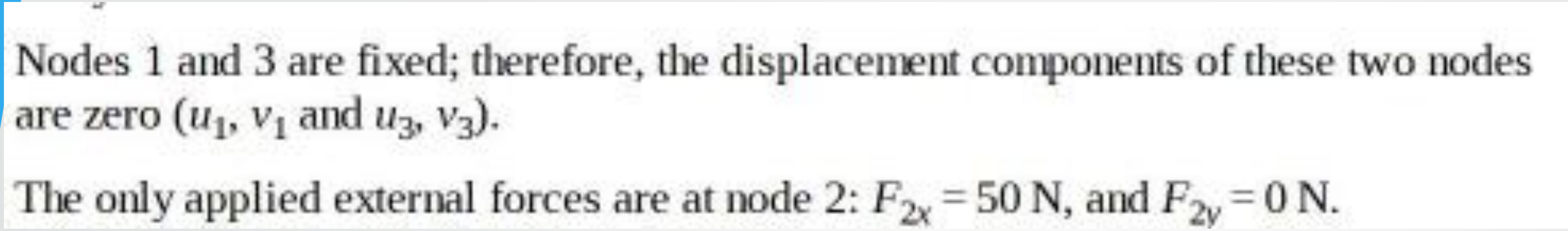


$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{2y}^{(2)} \\ f_{3x}^{(2)} \\ f_{3y}^{(2)} \end{Bmatrix} = 184{,}125 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}.$$

Global stiffness matrix,

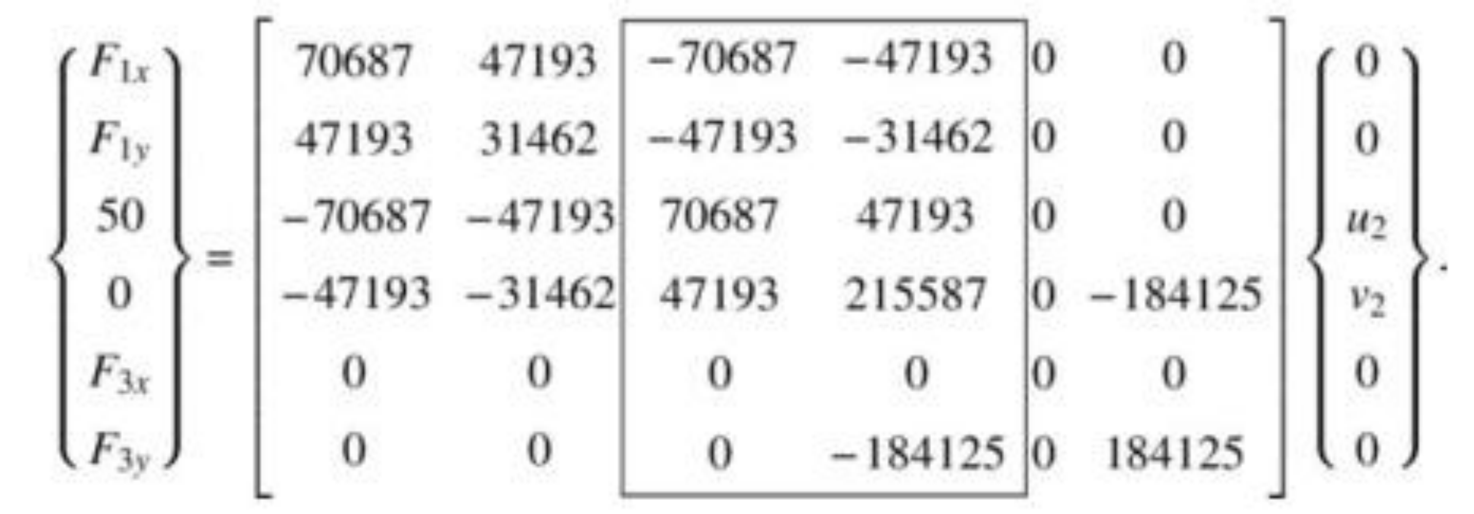


$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = \begin{bmatrix} 70687 & 47193 & -70687 & -47193 & 0 & 0 \\ 47193 & 31462 & -47193 & -31462 & 0 & 0 \\ -70687 & -47193 & 70687 & 47193 & 0 & \\ -47193 & -31462 & 47193 & 215587 & 0 & -184125 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -184125 & 0 & 184125 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}.$$



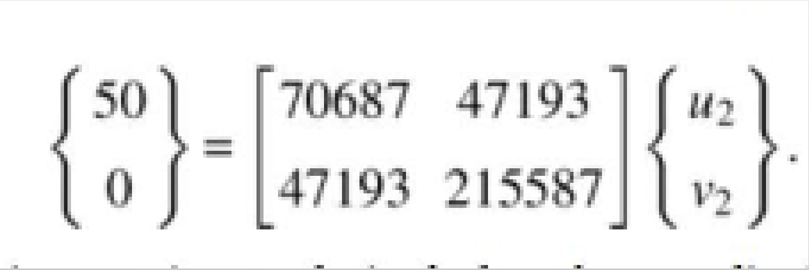
Nodes 1 and 3 are fixed; therefore, the displacement components of these two nodes are zero (u_1, v_1 and u_3, v_3).

The only applied external forces are at node 2: $F_{2x} = 50$ N, and $F_{2y} = 0$ N.



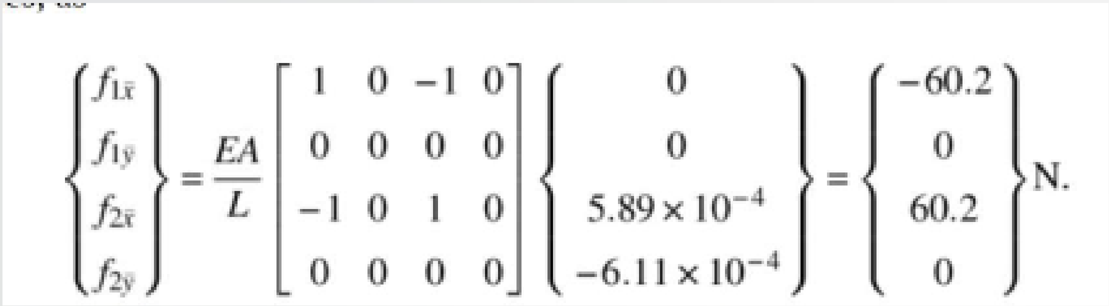
$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ 50 \\ 0 \\ F_{3x} \\ F_{3y} \end{Bmatrix} = \left[\begin{array}{cc|cc|cc} 70687 & 47193 & -70687 & -47193 & 0 & 0 \\ 47193 & 31462 & -47193 & -31462 & 0 & 0 \\ -70687 & -47193 & 70687 & 47193 & 0 & 0 \\ -47193 & -31462 & 47193 & 215587 & 0 & -184125 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -184125 & 0 & 184125 \end{array}\right] \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ 0 \end{Bmatrix}.$$

By using elimination approach ,



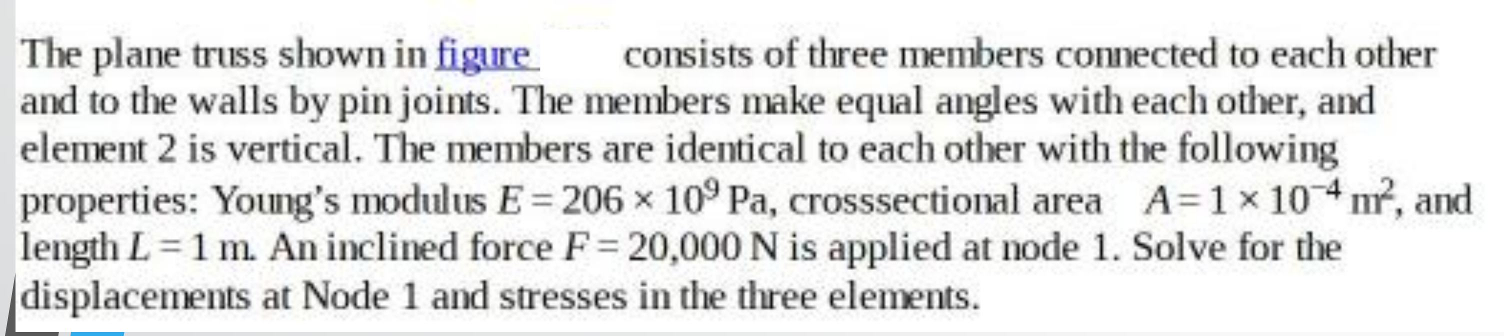
$$\begin{Bmatrix} 50 \\ 0 \end{Bmatrix} = \begin{bmatrix} 70687 & 47193 \\ 47193 & 215587 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix}.$$

For force calculation,

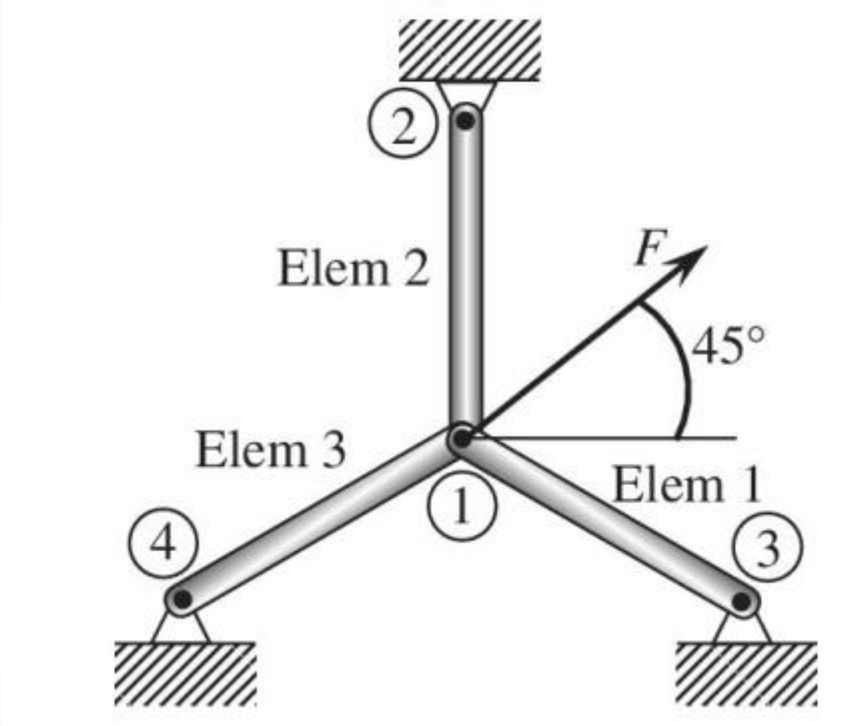


$$\begin{Bmatrix} f_{1\bar{x}} \\ f_{1\bar{y}} \\ f_{2\bar{x}} \\ f_{2\bar{y}} \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ 5.89\times 10^{-4} \\ -6.11\times 10^{-4} \end{Bmatrix} = \begin{Bmatrix} -60.2 \\ 0 \\ 60.2 \\ 0 \end{Bmatrix}\text{N.}$$

Problem:



The plane truss shown in figure consists of three members connected to each other and to the walls by pin joints. The members make equal angles with each other, and element 2 is vertical. The members are identical to each other with the following properties: Young's modulus $E = 206 \times 10^9$ Pa, crosssectional area $A = 1 \times 10^{-4}$ m^2, and length $L = 1$ m. An inclined force $F = 20{,}000$ N is applied at node 1. Solve for the displacements at Node 1 and stresses in the three elements.



Solution:

Element	AE/L	LN1 (i)	LN2 (j)	ϕ	$l = \cos\phi$	$m = \sin\phi$
1	206×10^5	1	3	$-\pi/6$	0.866	−0.5
2	206×10^5	1	2	$\pi/2$	0	1
3	206×10^5	1	4	$-5\pi/6$	−0.866	−0.5

To find the stiffness matrix of each element

$$\left\{\mathbf{f}^{(1)}\right\} = \left[\mathbf{k}^{(1)}\right]\left\{\mathbf{q}^{(1)}\right\}.$$

Where [k] is,

$$[k] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & \times C^2 & \times CS \\ & S^2 & \times CS & \times S^2 \\ & & C^2 & CS \\ \text{Symmetry} & & & S^2 \end{bmatrix}$$

For element 1

$$\left[\mathbf{k}^{(1)}\right] = 206 \times 10^5 \left[\begin{array}{cccc} 0.750 & -0.433 & -0.750 & 0.433 \\ -0.433 & 0.250 & 0.433 & -0.250 \\ -0.750 & 0.433 & 0.750 & -0.433 \\ 0.433 & -0.250 & -0.433 & 0.250 \end{array}\right]\begin{array}{c} u_1 \\ v_1 \\ u_3 \\ v_3 \end{array},$$

For element 2

$$\left[\mathbf{k}^{(2)}\right] = 206\times 10^5 \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \end{array},$$

For element 3

$$\left[\mathbf{k}^{(3)}\right] = 206\times 10^5 \left[\begin{array}{cccc} 0.750 & 0.433 & -0.750 & -0.433 \\ 0.433 & 0.250 & -0.433 & -0.250 \\ -0.750 & -0.433 & 0.750 & 0.433 \\ -0.433 & -0.250 & 0.433 & 0.250 \end{array}\right] \begin{array}{c} u_1 \\ v_1 \\ u_4 \\ v_4 \end{array}.$$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \\ F_{x3} \\ F_{y3} \\ F_{x4} \\ F_{y4} \end{Bmatrix} = 206\times 10^5 \begin{bmatrix} 1.5 & 0 & 0 & 0 & -0.750 & 0.433 & -0.750 & -0.433 \\ & 1.5 & 0 & -1 & 0.433 & -0.250 & -0.433 & -0.250 \\ & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 1 & 0 & 0 & 0 & 0 \\ & & & & 0.750 & -0.433 & 0 & 0 \\ & \text{Symmetric} & & & & 0.250 & 0 & 0 \\ & & & & & & 0.750 & 0.433 \\ & & & & & & & 0.250 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

$$F_{1x} = 20000 \cdot \cos(\pi/4) = 14,142$$
$$F_{1y} = 20000 \cdot \sin(\pi/4) = 14,142.$$

$$u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0.$$

$$206 \times 10^5 \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix} = \begin{Bmatrix} 14142 \\ 14142 \end{Bmatrix}.$$

$$u_1 = 0.458 \text{mm},$$
$$v_1 = 0.458 \text{mm}.$$

$$P^{(1)} = 206 \times 10^5 (0.866(u - u_1) - 0.5(v - v_1)) = -3,450 \text{ N}.$$

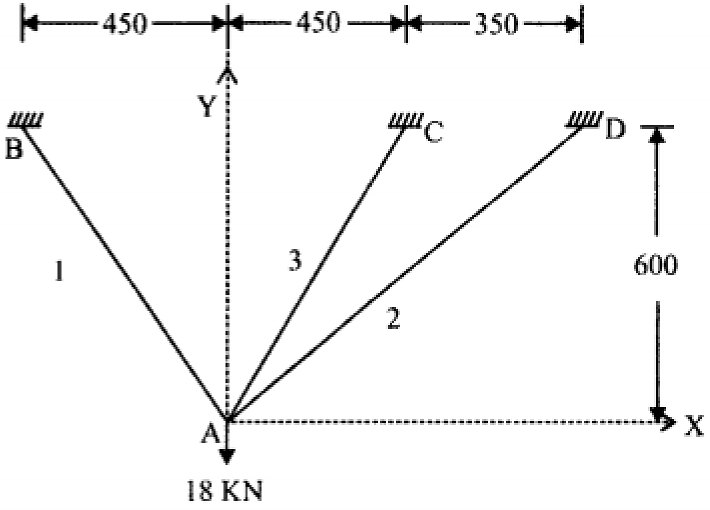
$$P^{(2)} = -9,440 \text{ N},$$
$$P^{(3)} = 12,900 \text{ N}.$$

$$\sigma^{(1)} = -34.5 \text{ MPa},$$
$$\sigma^{(2)} = -94.4 \text{ MPa},$$
$$\sigma^{(3)} = 129 \text{ MPa}.$$

Assignment 2

Problem No. 1 : Analysis the following system and compare answers with FEA software

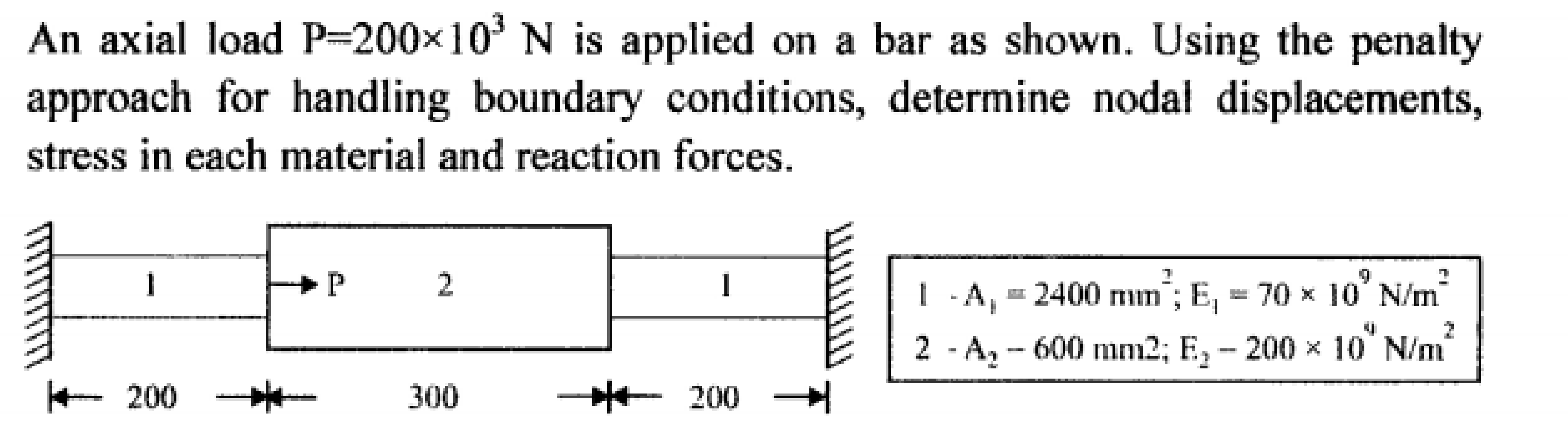
For the three bar truss shown in figure below, determine the displacements of node ‘A’ and the stress in element 3.



$A = 250\ mm^2$; $E = 200\ GPa$

Problem No. 2 : Analysis the following system. Compare answers with FEA software.

In FEA analysis following problem by taking 1D element and by 3Dmodelling the system



An axial load $P=200\times10^3$ N is applied on a bar as shown. Using the penalty approach for handling boundary conditions, determine nodal displacements, stress in each material and reaction forces.

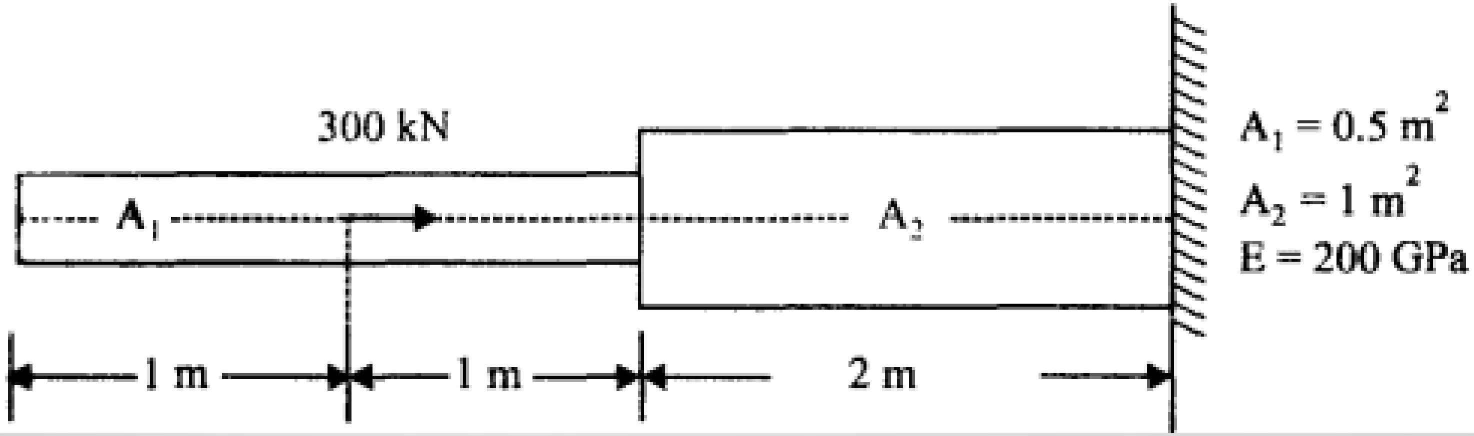
1 - A_1 = 2400 mm^2; E_1 = 70 × 10^9 N/m^2

2 - A_2 – 600 mm2; E_2 – 200 × 10^9 N/m^2

Problem No. 3 : Analysis the following system. Compare answers with FEA software.

In FEA analysis following problem by taking 1D element and by 3Dmodelling the system

Determine the nodal displacements and element stresses by finite element formulation for the following figure. Use P=300 k N; A_1=0.5 m^2; A_2=1 m^2; E=200 GPa



Thank You
For Your Attention